

# ON PATTERNS AND LANGUAGES IN 1-11-REPRESENTATIONS OF GRAPHS

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## EXTENDED ABSTRACT

( Consider for Best Paper Presentation. )

## Introduction

Representing graphs by words over their vertex sets provides a natural connection between graph theory and combinatorics on words. In such representations, adjacency is encoded via local patterns in words. A classical model in this direction is that of *word-representable graphs*, where two vertices are adjacent if and only if their corresponding letters alternate in the word. However, not all graphs admit such representations.

A natural generalisation of this concept is given by *1-11-representations*, where two vertices are adjacent if and only if the word restricted to those vertices contains at most one occurrence of consecutive identical letters. It has recently been shown that every graph admits a permutational 1-11-representation.

Repetition patterns play a central role in combinatorics on words. A *square* (respectively, *cube*) is obtained by repeating a nonempty word twice (respectively, three times) consecutively. It is known that every word-representable graph admits a cube-free representation, while square-free representations exist for all such graphs except the empty graph on two vertices. Since not all graphs are word-representable, it is natural to investigate whether more general representations, such as 1-11-representations, can avoid such repetitions.

From a formal language perspective, the set of all words representing a given word-representable graph forms a regular language. Since 1-11-representations generalise word-representations, a natural question is whether the language of all 1-11-representations of a graph is also regular.

## Main Results

We now present our main results on repetition patterns and language properties of 1-11-representations.

To understand whether cubes can be avoided in optimal representations, we first consider the following example.

**Theorem 1.** *Let  $G$  be the graph consisting of a complete graph  $K_3$  on vertices  $\{1, 2, 3\}$  together with an isolated vertex  $v$ . Then every minimum-length 1-11-representation of  $G$  necessarily contains a cube.*

To analyse cube structures in permutational representations, we first establish structural restrictions.

**Lemma 1.** *Let  $w$  be a permutational 1-11-representation of a graph on  $n$  vertices. Then  $w$  does not contain a cube  $XXX$  where the length of  $X$  is not a multiple of  $n$ .*

To simplify representations, we show that redundant permutation blocks can be removed.

**Lemma 2.** *If a permutational 1-11-representation contains two consecutive identical permutations, then one occurrence can be removed without changing the represented graph.*

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As a consequence, cubes formed by identical permutations are not essential.

**Corollary 1.** *If a permutational 1-11-representation contains a cube consisting of identical permutations, then two copies can be removed without affecting the graph.*

We now show that cubes can always be eliminated in the permutational setting.

**Theorem 2.** *If a permutational 1-11-representation of a graph contains a cube, then there exists a cube-free permutational 1-11-representation of the same graph.*

This immediately yields a property of optimal permutational representations.

**Corollary 2.** *If a graph has a permutational 1-11-representation number  $k$ , then every representation consisting of  $k$  permutations is cube-free.*

We next investigate whether similar results hold for squares.

**Theorem 3.** *Let  $G$  be the graph consisting of a complete graph  $K_3$  together with an isolated vertex  $v$ . Then every permutational 1-11-representation of  $G$  with the minimum number of permutations necessarily contains a square.*

Finally, we study these representations from a language-theoretic perspective.

**Theorem 4.** *The language of all 1-11-representations of a given graph  $G(V, E)$  is regular.*

This extends naturally to the permutational setting.

**Corollary 3.** *The language of all permutational 1-11-representations of a given graph  $G(V, E)$  is regular.*

## Conclusion

In this paper, we studied repetition patterns in 1-11-representations of graphs and showed that cube-free representations can always be obtained in the permutation framework, while square-free representations may be unavoidable. We also established that the language of all 1-11-representations of a given graph, as well as the language of all permutational 1-11-representations, is regular, highlighting a strong connection between graph representations and formal language theory.

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